# Gluon production and diffraction in the dipole picture

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Using the eikonal approximation, we show that inclusive gluon production is related to the scattering of a gluon-gluon dipole while diffractive gluon production in DIS is related to a two-$q\bar{q}$ dipole scattering amplitude. Hence diffractive photon dissociation cross-sections are observables that provide potential information on dipole correlations, which represent an open issue in high-energy QCD.

## 1. INTRODUCTION

Over the past ten years, the perturbative $q\bar{q}$ dipole picture has been developed [ 1] with the aim of understanding high-energy scattering in QCD. This formalism is well-suited to study scattering near the unitarity limit because density effects and non-linearities that lead to saturation and unitarization of the scattering amplitudes can be taken into account. The dipole picture has had great succes for the phenomenology of hard processes initiated by virtual photons, in which case the link between the experimental probes and the scattering of $q\bar{q}$ dipoles is established. This is not the case for hadron-hadron collisions in which the probes are the final-state particules that one measures, *e.g.* a final-state gluon that we detect as a jet. Establishing a link between the scattering of dipoles and observables like jet cross-sections is of great theoretical and phenomenological interest for the Tevatron and in the prospect of the LHC.

In the purpose of doing so, we derive the cross-sections for inclusive and diffractive gluon production in the high-energy scattering of a $q\bar{q}$ dipole off an arbitrary target and show that they are related to the scattering of dipole configurations. In the inclusive case, the relevant object is a gluon-gluon dipole ($gg$ dipole) and one can show this is the case independently of the incident projectile. In the diffractive case we find that two-$q\bar{q}$ dipole amplitudes are involved; this provides a link between observables like diffractive photon dissociation cross-sections measurable at HERA and correlations between dipoles.

## 2. HIGH-ENERGY EIKONAL SCATTERING

Let us start with the eikonal approximation for quarks and gluons scattering at high energies. When a system of partons propagating at nearly the speed of light passes through a target and interacts with its gauge fields, the dominant couplings are eikonal: the partons have frozen transverse coordinates and the gluon fields of the target do not vary during the interaction. This is justified since the time of propagation through the

target is much shorter than the natural time scale on which the target fields vary. The effect of the interaction with the target is that the partonic components of the incident wavefunction pick up eikonal phases: if $|(\alpha,x)\rangle$ (resp. $|(a,x)\rangle$) is the wavefunction of an incoming quark of color $\alpha\in[1,N_c]$ (resp. gluon of color $a\in[1,N_c^2-1]$) and transverse position $x$ (the irrelevant degrees of freedom like spins or polarizations are not explicitly mentioned), then the action of the $\mathcal{S}-$matrix is (see for example [ 2]):

$$\mathcal{S}|(\alpha,x)\rangle\otimes|t\rangle=\sum_{\alpha'}\left[W_F(x)\right]_{\alpha\alpha'}|(\alpha',x)\rangle\otimes|t\rangle\ ,\quad \mathcal{S}|(a,x)\rangle\otimes|t\rangle=\sum_b W_A^{ab}(x)|(b,x)\rangle\otimes|t\rangle\ ,\ (1)$$

where $|t\rangle$ denotes the initial state of the target. The phase shifts due to the interaction are the color matrices $W_F$ and $W_A$, the eikonal Wilson lines in the fundamental and adjoint representations respectively, corresponding to propagating quarks and gluons. They are given by

$$W_{F,A}(x)=\mathcal{P}\exp\{ig_s\int dz_+T^a_{F,A}\mathcal{A}^a_-(x,z_+)\} \tag{2}$$

with $\mathcal{A}_-$ the gauge field of the target and $T^a_{F,A}$ the generators of $SU(N_c)$ in the fundamental ($F$) or adjoint ($A$) representations. We use the light-cone gauge $\mathcal{A}_+=0$ and $\mathcal{P}$ denotes an ordering in the light-cone variable $z_+$ along which the incoming partons are propagating.

For an incoming state $|\Psi_{in}\rangle$, the outgoing state $|\Psi_{out}\rangle=\mathcal{S}|\Psi_{in}\rangle\otimes|t\rangle$ emerging from the eikonal interaction is obtained by the action of the $\mathcal{S}-$matrix on the partonic components of $|\Psi_{in}\rangle$ as indicated by formula (1). The outgoing wavefunction $|\Psi_{out}\rangle$ is therefore a function of the Wilson lines (2). When calculating physical observables from $|\Psi_{out}\rangle$, one obtains objects that are target averages of traces of Wilson lines (the traces come from the color summations that one has to carry out). As an example, the simplest of these objects is

$$S_{q\bar{q}}(x,x')=\frac{1}{N_c}\left\langle\mathrm{Tr}\Big(W_F^\dagger(x')W_F(x)\Big)\right\rangle_t \tag{3}$$

where we have denoted the target averages $\langle t|\ .\ |t\rangle=\langle\ .\ \rangle_t$. This is the $q\bar{q}$ dipole elastic $\mathcal{S}-$matrix ($x$, $x'$ : positions of the quark and antiquark) which enters for example in the DIS total cross-section; more generally, observables are functions of (3) or more complicated amplitudes. To compute these amplitudes, one has to evaluate the averages $\langle\ .\ \rangle_t$ which amounts to calculating averages of Wilson lines in the target wavefunction. A lot of studies are devoted to this problem, here we only establish the link between the observables and the dipole amplitudes as we shall do now with gluon-production cross-sections.

## 3. INCLUSIVE AND DIFFRACTIVE GLUON PRODUCTION

Let us consider that the incoming state is an incident $q\bar{q}$ dipole of transverse size $r_0$ ($|r_0|\ll 1/\Lambda_{QCD}$ in order to justify the use of perturbation theory), see Figure 1. The inclusive cross-section for the production of a gluon of transverse momentum $k$ and rapidity $y$ in the scattering of this $q\bar{q}$ dipole off an arbitrary target then reads [ 3]

$$\frac{d\sigma_{incl}}{d^2kdy}(r_0)=\frac{4\alpha_sC_F}{\pi k^2}\int_0^{|r_0|}dz\ J_0(kz)\log\frac{|r_0|}{z}\ \frac{\partial}{\partial z}z\frac{\partial}{\partial z}\int d^2b\Big(1-S_{gg}\Big(b+\frac{z}{2},b-\frac{z}{2}\Big)\Big)\ , \tag{4}$$

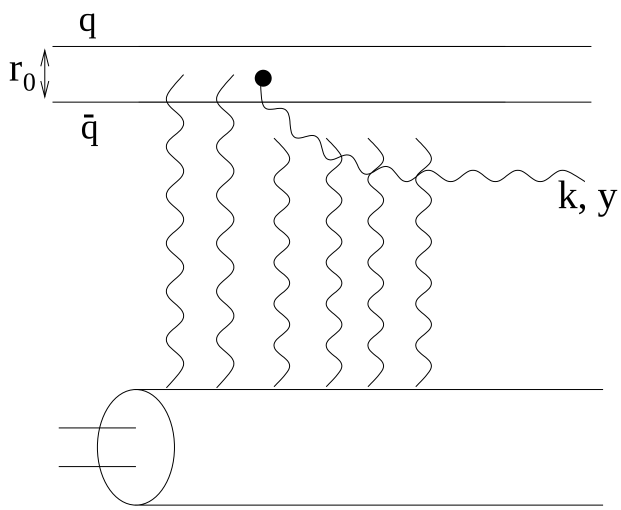


Figure 1. Gluon production off a $q\bar{q}$ dipole of size $r_0$. $k$ and $y$ : transverse momentum and rapidity of the measured gluon.

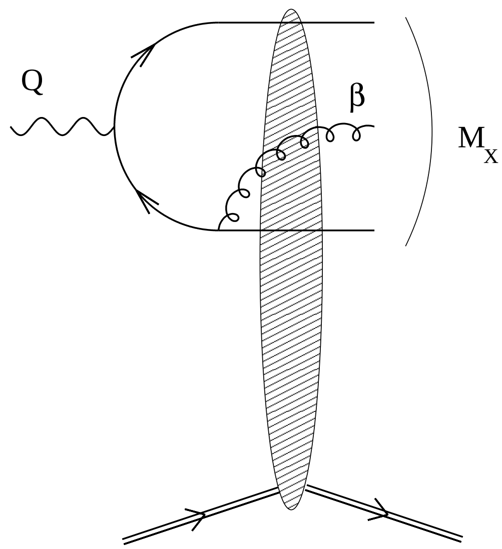


Figure 2. Diffractive dissociation of a photon of virtuality $Q$. $M_X$ : mass of the final state. $y = \log 1/\beta$.

where $S_{gg}$ is $\mathcal{S}-$matrix for the scattering of a $gg$ dipole on the target and is given by

$$S_{gg}(x,x') = \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\Big(W_A^\dagger(x')W_A(x)\Big)\right\rangle_t \ . \tag{5}$$

The azimutal angle of the incoming $q\bar{q}$ dipole has been integrated out so the cross-section depends only on $|r_0|$. The diffractive cross-section for gluon production reads [ 3] (we define the diffractive process as one in which a color singlet is exchanged and the target does not break up):

$$\frac{d\sigma_{diff}}{d^2kdy}(r_0) = \frac{\alpha_s N_c^2}{4\pi^2 C_F}\int d^2b\ \mathbf{A}\Big(k, b+\frac{r_0}{2}, b-\frac{r_0}{2}\Big).\mathbf{A}^*\Big(k, b+\frac{r_0}{2}, b-\frac{r_0}{2}\Big) \tag{6}$$

where the two dimensional vector $\mathbf{A}$ is given by

$$\mathbf{A}(k,x,x') = \int \frac{d^2z}{2\pi}\ e^{-ik.z}\left[\frac{z-x}{|z-x|^2} - \frac{z-x'}{|z-x'|^2}\right]\Big(S_{q\bar{q}}^{(2)}(x,z;z,x') - S_{q\bar{q}}(x,x')\Big). \tag{7}$$

$$S_{q\bar{q}}^{(2)}(x,z;z',x') = \frac{1}{N_c^2}\left\langle \mathrm{Tr}\Big(W_F^\dagger(x')W_F(z')\Big)\mathrm{Tr}\Big(W_F^\dagger(z)W_F(x)\Big)\right\rangle_t \tag{8}$$

is the $\mathcal{S}-$matrix for the scattering of two $q\bar{q}$ dipoles on the target. To write down these cross-sections, it is assumed that the measured gluon is soft, that is we are working in the leading logarithmic approximation in $1/\beta \equiv e^y$. The $\mathcal{S}-$matrices involved in those formulae contain the scatterings with all numbers of gluon exchanges with the target and, via the quantum evolution of the target, they also contain the emissions of gluons softer than the measured one $(k,y)$. Then if $Y$ is the total rapidity, the $\mathcal{S}-$matrices depend on $Y-y$.

The result for the inclusive cross-section (4) can be extended to an arbitrary projectile [ 3] if its natural scale $1/r_0$ is much smaller than $k$, the result being valid at double leading logarithmic accuracy. This is of great importance in the prospect of jet cross-sections at the LHC. For the diffractive cross-section (6), the same extension does not appear to be possible, this could be related to the breakdown of collinear factorization for diffractive cross-sections.

## 4. DIFFRACTIVE PHOTON DISSOCIATION

The most straightforward application to our result for the diffractive cross-section is the diffractive photon dissociation at large mass in DIS: a photon of virtuality $Q^2$ scatters on a target proton which stays intact, see Figure 2. If the final-state diffractive mass is large enough, the cross-section is dominated by the $q\bar{q}g$ component and is given by our result convoluted with the photon-to-$q\bar{q}$ dipole wavefunction $\phi^\gamma$:

$$\frac{d\sigma^\gamma_{diff}}{d^2k\ dM_X} = \frac{2}{M_X}\int d^2r_0\ \phi^\gamma(|r_0|,Q^2)\ \frac{d\sigma_{diff}}{d^2kdy}(r_0)\ . \tag{9}$$

Phenomenology has been done [ 4] for the cross-section integrated over $k^2$ neglecting the correlations (i.e. writting $S^{(2)}_{q\bar{q}}=S^2_{q\bar{q}}$). Taking them into account might be a difficult task but seems necessary in order to gain insight on dipole correlations from the observable (9) and the related integrated cross-sections.

As an example, the recent treatment of correlations in [ 5] allows a first application. Neglecting non-dipole amplitudes in the complicated system of equations verified by the $\mathcal{S}-$matrices in the high-energy limit, the authors found a reduction of the problem: $1-S^{(2)}_{q\bar{q}}(x,z;z',x')=c(N(x,z)+N(z',x')-N(x,z)N(z',x'))$ and $1-S_{q\bar{q}}(x,x')=cN(x,x')$ where $0<c<1$ is an arbitrary number mesuring the strength of the correlations and where $N$ is solution of a closed equation called the BK equation [ 6]. One has then for example

$$\sigma^\gamma_{diff} = c^2\ \sigma^\gamma_{diff}\Big|_{BK} \tag{10}$$

where $c$ is put to 1 in $\sigma^\gamma_{diff}|_{BK}$. This quantity is fully computable from a solution of the BK equation. The value of $c$ could then be extracted from experiments.

Precise measurements of $\sigma^\gamma_{diff}$ should be made to try and take advantage of this link between large-mass diffraction and dipole correlations while deeper theoretical studies are being carried out.

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